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**UTILIZATION OF FAST RUNNING MODELS IN BURIED BLAST SIMULATIONS
OF GROUND VEHICLES FOR SIGNIFICANT COMPUTATIONAL EFFICIENCY**

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ABSTRACT

Over the course of typical survivability analyses for underbody blast events, a multitude of individual cases are examined where charge size, charge location relative to the vehicle, and vehicle clearance from the ground are varied, so as to arrive at a comprehensive assessment. While multi-physics computational tools have reduced the expense and difficulty of testing each loading case experimentally, these tools still often require significant execution and wall-clock times to perform the simulations. In efforts to greatly reduce the time required to conduct a holistic survivability analysis, Fast Running Models (FRMs) have been implemented and validated to act as a surrogate for the computationally expensive finite element tools in use today. Built using a small set of simulations, FRMs generate loading data in a matter of seconds, representing a significant improvement in survivability analysis turnaround time.

INTRODUCTION

Survivability assessment comprises an integral part of system analysis for any Army vehicle [1]. A particular method of survivability assessment that is often used is achieved in two steps. First, pressure loads on the vehicle arising from detonation of an explosive are calculated. The pressure loads are then applied to the finite element model of the vehicle during the second step, in order to determine the structural response of the vehicle, including damage, if any. The accelerative forces that are imparted to vehicle occupants can also be computed if Anthropomorphic Test Device (ATD) dummy models are included in the simulations. Alternatively, the forces arising from the pressure loads can be applied to a rigid multi-body dynamics model of the vehicle in order to evaluate the dynamic response of the vehicle to the underbody blast.

Different organizations within the Army apply the blast loads to either high-fidelity finite element models or to rigid-body dynamic vehicle models, depending on the focus of the analysis. For either application, the simulations involving fluid-structure interactions used to determine blast loading are highly time-consuming. Holistic survivability assessments must determine vehicle and occupant responses to explosive charges of varied size and placement, while also considering different levels of vehicle ground clearance. Performing high-fidelity, computationally expensive analyses for myriad load evaluations is not optimal, hence the motivation for developing a new reduced-order modeling approach for the computation of blast loading.

A Fast Running Model (FRM) methodology is presented and validated in this paper. The development of the FRM is based on theoretical formulations presented originally in [2,3] and later refined in [4-6]. Development of the FRM requires conducting complete simulations at a limited number of training points (each training point corresponds to a different explosive placement

configuration). These results are then used to construct an FRM that can rapidly predict the effects of an explosive configuration that was not included in the original training point set. Utilization of FRMs yields substantial time savings, and creates the opportunity for a much more thorough and systematic exploration of the design space.

METHODS

In this work, an FRM is built through integration of both Principal Component Analysis (PCA) [7-9] and the Kriging method for metamodel generation [10, 11]. PCA distills the blast loading time histories generated by actual simulations (at the training points) into “modal” type information. The term “modal” does not imply that there are linear limitations in any way. It is utilized as an indication that the time domain histories are decomposed into more fundamental properties which provide a mathematical expansion basis for the time histories.

The central concept of PCA is to reduce the dimensionality of a data set characterized by a large number of interrelated variables, while retaining the maximum possible quantity of information in the data set. Principle component analysis has been applied to non-linear structural analysis in [12,13], but has also been used in fields as wide-ranging as image processing, immunology, and molecular dynamics [14]. PCA has also provided an expansion basis for automotive crash analyses [15, 16], occupant safety applications [5], shock analysis [4], and driveline performance simulations [17]. PCA requires a response matrix at the outset. This matrix contains time series data for model responses at different locations, and it can represent pressure, velocity, displacement, etc. The response matrix X is given by:

$$X = \begin{bmatrix} x_1(t_1) & \dots & x_1(t_k) \\ \vdots & \ddots & \vdots \\ x_j(t_1) & \dots & x_j(t_k) \end{bmatrix} \tag{1}$$

where each row corresponds to a different location on the model and each column corresponds to a different time step. It is assumed that $J \leq K$, since the number of time steps in the simulation is highly likely to exceed the number of locations being monitored. Intrinsic to each time history are the non-linear phenomena present for underbody blast, including shock waves, venting, and reflection. The decomposition of response matrix X is performed using the following equation:

$$X = USV^T \tag{2}$$

where U is a column-orthogonal $J \times J$ matrix, S is a diagonal $J \times J$ matrix, and V^T is an orthogonal $J \times K$ matrix.

The columns of U comprise the left singular vectors, the rows of V^T contain the elements of the right singular vectors, and only r number of elements on the diagonal of S are zero. The left singular vectors are the eigenvectors of XX^T . By convention, the ordering of the singular vectors is determined by high-to-low sorting order of singular values, with the highest singular value in the upper left position of the S matrix. Conceptually, matrix U represents the 'modal' information in the response matrix, W contains energy information for each mode in U , and V^T contains time domain information that is used to reproduce time histories. The nature of the decomposition matrices is expressed in figure 1.

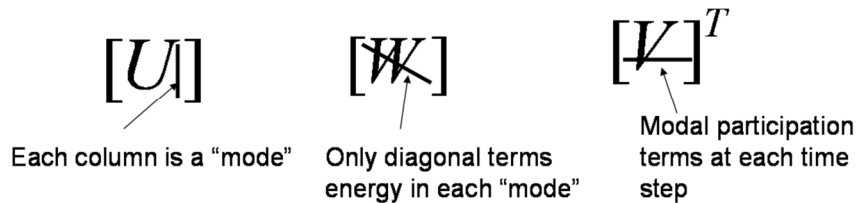


Figure 1: Principal component analysis decomposition matrices.

In order to illustrate the physical meaning of the “modes” computed by the PCA, the first two “modes” for a quarter of a simply supported plate loaded by a central impact force are shown in figure 2. PCA is useful for non-linear models for the same reason that modal properties are useful in linear dynamic systems; it provides an expansion basis for representing the response of the system through a reduced basis. The PCA provides a compact representation of the response for a non-linear model. It can be

observed in figure 2 that the “modal” information contained in each one of the columns of the [U] matrix increases in complexity for higher modes and that each “mode” satisfies the physical boundary conditions.

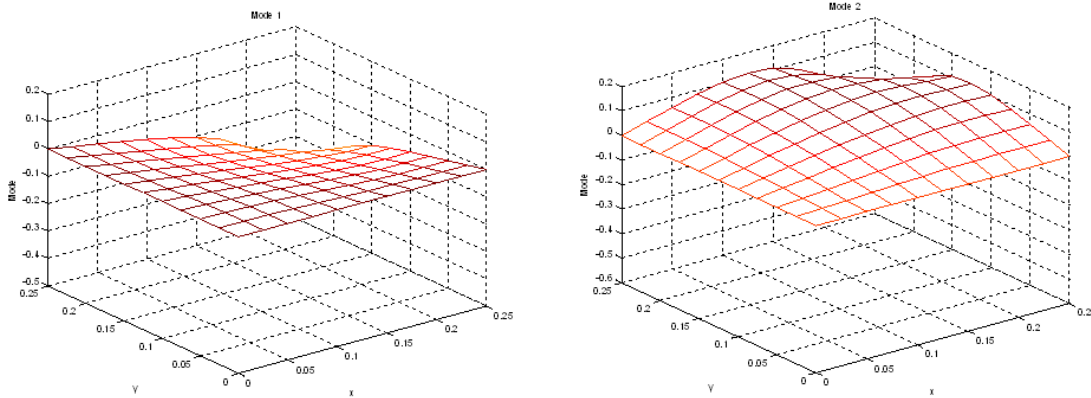


Figure 2: First and second modes for one quadrant of a simply supported plate excited by an impact load at the center of the plate.

Each matrix on the right hand side of equation (2) is partitioned based on the relative importance of the singular values contained in the diagonal entries of matrix S :

$$X = [\Phi \quad \Phi_\tau] \begin{bmatrix} D & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} \eta \\ \eta_\tau \end{bmatrix}. \quad (3)$$

Therefore, S is partitioned into four blocks, and U and V^T are partitioned accordingly, and:

$$X = \Phi D \eta. \quad (4)$$

D is a $p \times p$ diagonal matrix, where p is the number of dominant singular values. The solution of the p number of singular values that are considered dominant is based on the relative energy contained in each nonzero singular value. The columns contained in Φ and the rows contained in η are called principle components, and the principle component decomposition is defined as:

$$X = \Phi D \eta + \Phi_\tau Z \eta_\tau \cong \Phi D \eta. \quad (5)$$

Therefore, the contribution of Φ_τ and η_τ to the definition of X is considered to be negligible.

The Kriging method is used to generate metamodels for the principal components of the training point simulations in order to build FRMs. Kriging asserts that the response at a particular location can be determined by considering the variability in the parameters of the numerical model. Denoting the parameter set as γ , equation (5) is transformed into equation (6) below.

$$[X(\gamma)] = [\Phi(\gamma)][D(\gamma)][\eta(\gamma)] \quad (6)$$

$[\Phi(\gamma)]$, $[D(\gamma)]$, and $[\eta(\gamma)]$ are evaluated from metamodels that are generated using the Kriging method [11,18-20].

To build an FRM, a response matrix X must be constructed in order to establish the mathematical relationship between input parameters and the response. The rows of the response matrix are time series data from individual computational simulations. When more simulations are used, the metamodel is improved as the correlations are fine-tuned. Because the computational simulations can be expensive (which is, after all, the reason to pursue FRMs), these seed runs, or training points, must be kept to a minimum. In order to maximize the benefit from each simulation, the training points must be carefully selected. An Optimal Symmetric Latin Hypercube (OSLH) algorithm has been implemented in order to populate the training space as fully as possible [22].

APPLICATION

FRMs have the potential to accelerate analysis efforts for any physics-based process that is traditionally modeled with a computationally-expensive numerical tool. One such tool is LS-DYNA, a commercial off-the-shelf (COTS) finite element program commonly used to analyze underbody blast scenarios for Army vehicles. In a typical analysis case, several design parameters are varied in order to probe the full breadth of possible blast loading effects on a vehicle. Charge weight, placement with regard to the footprint of the vehicle, depth of burial, and vehicle ground clearance have been identified as parameters that are typically varied. The sampling points, therefore, are unique combinations of these parameters that evenly fill the design space.

Once the sampling points have been determined, LS-DYNA simulations must be run for each configuration. The results are then used to construct the response matrix that is used to form an FRM. Principle component decomposition is performed and Kriging models are constructed for the principle components of the sample point time series that comprise the response matrix. When a response is desired for parameter combinations that are not included in the training point set, the FRM is queried and results are generated in a matter of seconds. It is common for a vehicle design team to test several combinations of the design parameters; FRMs enable the completion of such studies in the time required to conduct the training point simulations (FRMs can be built from training point results in seconds. Once built, FRMs generate results in seconds as well). As long as the number of training points is less than the number of evaluation points, or the parameter combinations of interest not included in the training point set, time is saved by using FRMs.

There is a second advantage that emerges from the use of FRMs as surrogates for computational expensive high-fidelity methods. It is plausible that supercomputing resources may be unavailable for quick-turnaround results. At other times, clusters may be idle. FRMs provide an opportunity to use these systems with much more efficiency. New training point responses can be collected when the supercomputer is idle, refining the FRM with no detriment to other computing tasks. When quick results are desired, and the computing resources are unavailable due to large job queues or system maintenance, the refined FRM can be called upon to provide instant results. With this approach, FRMs provide a means to smooth the boom-and-bust cycle of computational resource availability.

BEST METHODOLOGY

The Blast Event Simulation sysTem (BEST) has been used for the training point simulations that are necessary to construct an FRM [22-24]. The LS-DYNA simulations to generate the FRM do not employ the typical, computationally expensive Arbitrary Lagrangian-Eulerian approach for the full run. Instead, the faster method implemented in the BEST tool has been included [22, 23]. The BEST approach treats the detonation and structural response of an underbody blast simulation as separate analysis phases. This is possible because the blast pulse and the structural response occur at different time intervals. In short, the structural model is used to define the upper profile of a purely Eulerian mesh. Elements on the upper boundary are constrained as if the vehicle was actually located directly above them. Pressure histories are recorded on these elements during a detonation simulation, and then can be applied to the vehicle underbody during a purely Lagrangian simulation.

The two-step BEST approach has been previously validated for the response of a structure with a V-shaped bottom [24]. Response of an occupant inside the structure was also examined as a part of that study. To further validate the BEST method, results using this process have been compared to physical experiments at the Aberdeen Proving Grounds (APG), Maryland. The experiments used below [26] form a benchmark that is being used to assess numerical means of predicting blast effects, so they serve as a good validation test case for BEST as well. Three geometries were included in the test, as seen in figure 3, and the burial depth of the charge was 50 mm.

Stereographic cameras were used to track the displacement of the structures in this set of experiments. These measured displacement data were used to first calculate the velocities and then the impulses using simple equations of physics. Experimental results were compared with both full LS-DYNA/ALE as well as BEST simulation results for geometries of 10, 20, and 30 degree angles at the 600 g charge weight, with an additional test case with 800 g charge weight used for the 20 degree geometry. The comparisons can be seen in figure 4.

While these are emerging results that are still in the process of being fully analyzed, it appears that the BEST predictions exhibit a correlation with experimental data that is, at minimum, as strong as the ALE simulations and, typically, stronger. The BEST results were also produced faster than the ALE results because the Eulerian and Lagrangian analyses have been separated. The test data in figure 4 add to the validation of the BEST tool, undergirding its inclusion here in the process used to create FRMs.

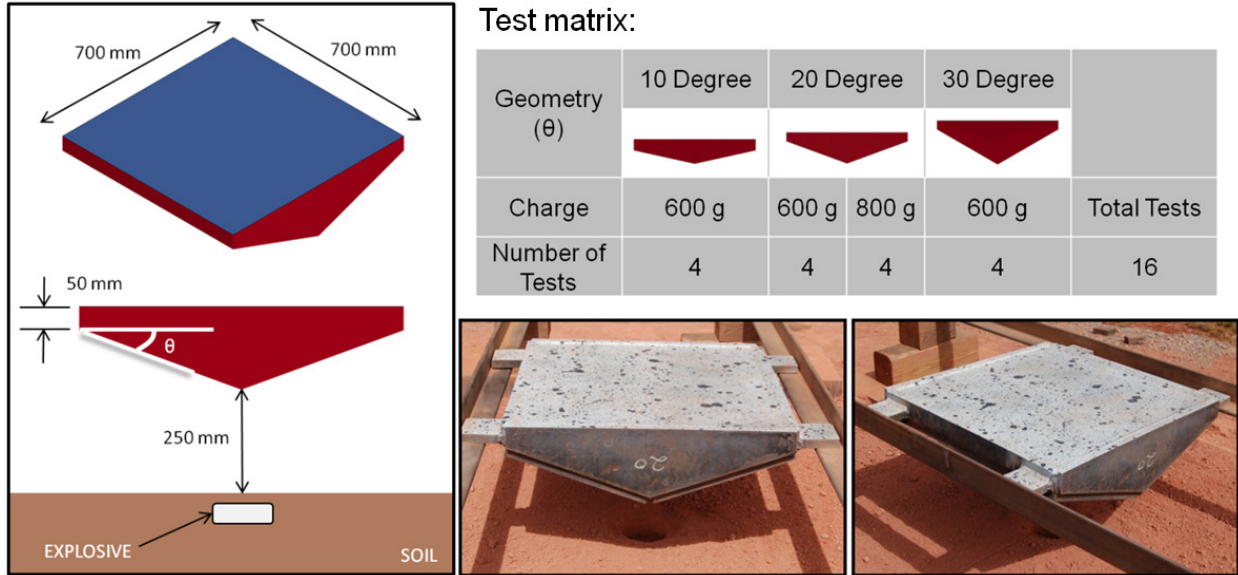


Figure 3: Experimental setup for benchmark cases.

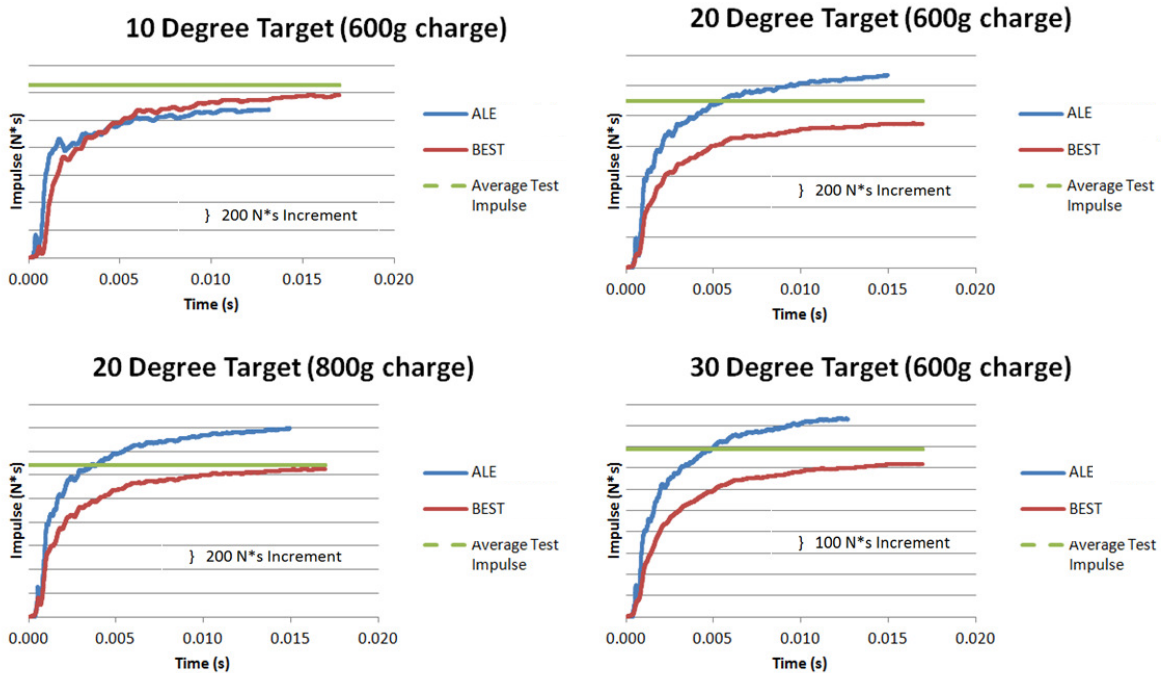


Figure 4: Impulse-time histories for benchmark v-hull geometries.

GRAPHICAL USER INTERFACE

An interface has been developed within the Blast Event Simulation system (BEST) to aid design engineers in building and using FRMs quickly, consistently, and in an organized manner. Some figures in the following Results section will be snapshots of the interface that has been developed. To use FRMs as surrogates for sophisticated multi-physics tools, design engineers must first build the FRM using the top panel in figure 5. Constructing an FRM entails specifying a range of interest, generating training point

data, and adding specific loading parameters governing the application of blast loads modeled with the FRM. After an FRM has been built, it can be accessed using the bottom panel in figure 5, where desired evaluation cases are specified.

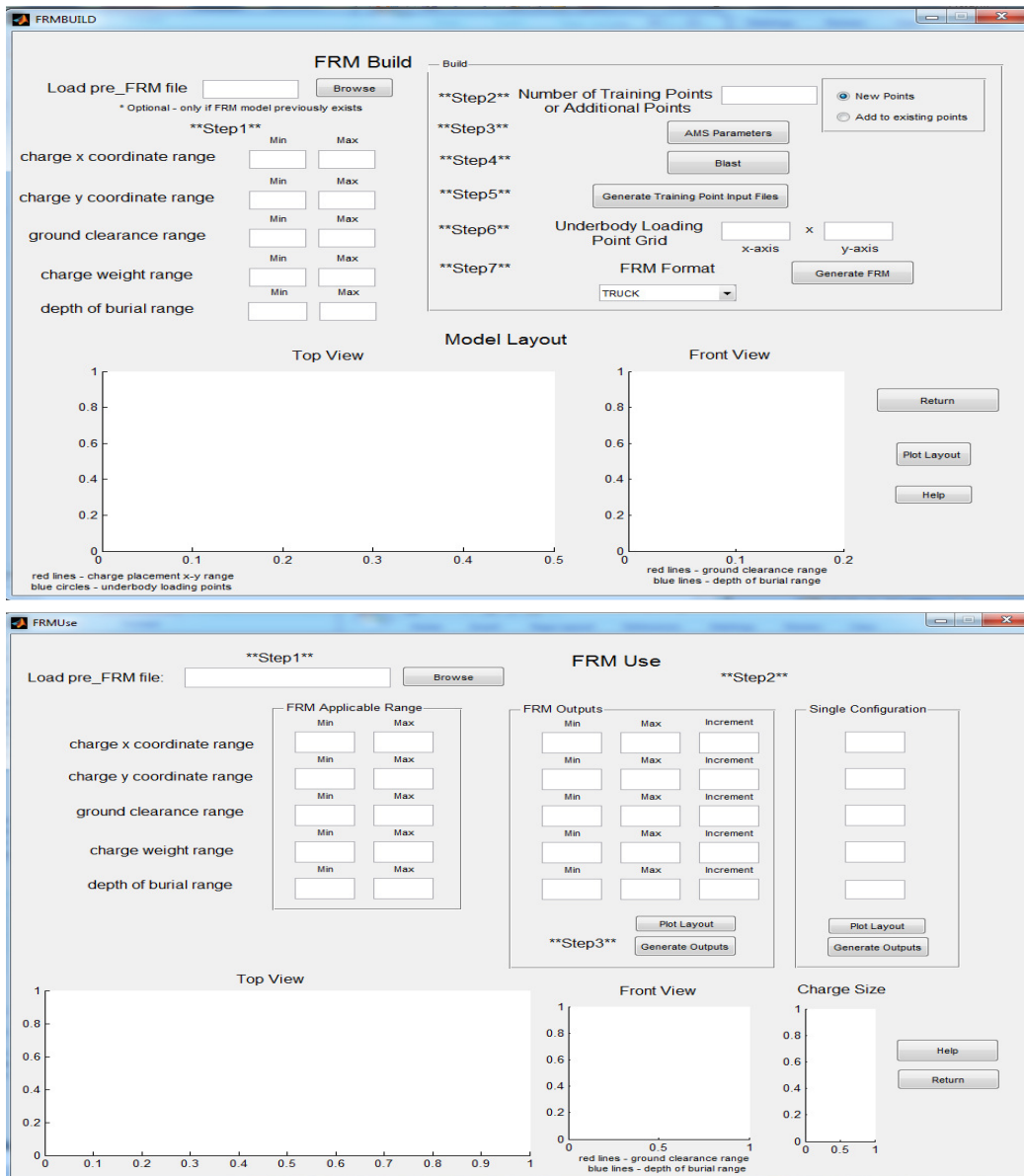


Figure 5: FRM Build and FRM Use panels for constructing and querying Fast-Running Models.

The FRM Build panel prompts the user to specify the range of variation for each design parameter (charge x coordinate range, charge y coordinate range, ground clearance range, charge weight range, and depth of burial range) and then the number of training points that are desired. The FRM Build panel is capable of launching other subpanels where an engineer can specify the details of the LS-DYNA simulations, such as mesh size, density, simulation duration, soil model [25] etc. Once simulation parameters have been specified, the FRM Build panel automatically generates LS-DYNA input files for each training point simulation.

Once the user has completed the BEST simulations for each FRM training point, they are asked to specify the loading point grid. Loading points denote the discretization of the vehicle underbody where unique pressure loads are calculated. Structural elements

are discretized into distinct regions based on proximity to these loading points. When the FRM outputs a pressure history for each loading point, the pressure histories associated with each loading point are applied to their respective "tributary" discretized regions. As will be seen in the Results section, the number of unique pressure histories at each evaluation point is equal to the number of loading points used to build the FRM. A TARDEC general purpose V-Hull used in the case study and the discretized loading regions for a 3x3 loading point grid (that is, 9 loading points, and 9 tributary regions corresponding to each loading point) are shown in figure 6. The loading point grid can be seen in figure 7, in the Results section.

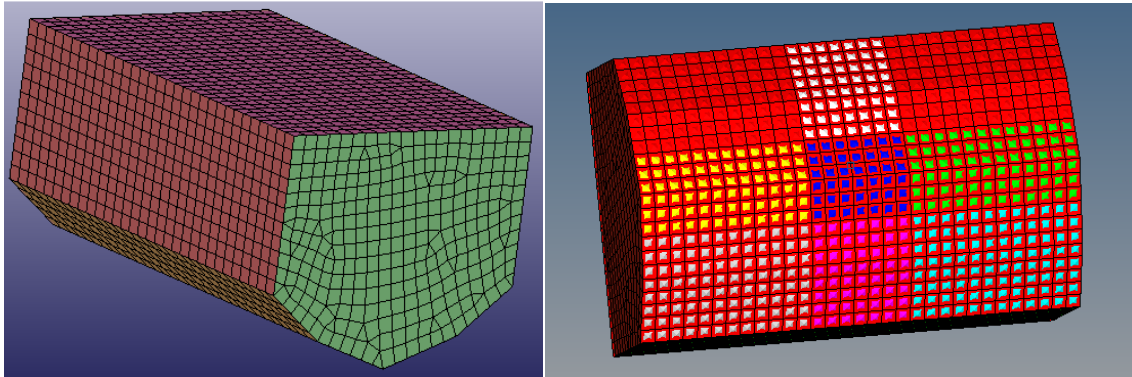


Figure 6: TARDEC V-Hull and loading regions on V-Hull underbody for a 3x3 loading point grid.

With the training point simulations and the loading point grid specified, the FRM Build panel can be used to construct an FRM. Once the FRM is built, the tool provides a visual representation of the loading point grid as well as the x and y ranges of charge placement, depth of burial range, and ground clearance range for which the FRM is applicable. This will be demonstrated in the following section.

RESULTS

An FRM is created for the pressures applied to the exterior of the TARDEC V-Hull during an underbody blast event. Twenty training point simulations comprising unique configurations of explosive placement and ground clearance are used to create an FRM for the blast loading. The ranges of variation for transverse and longitudinal mine placement are 0.7m and 1m, respectively.

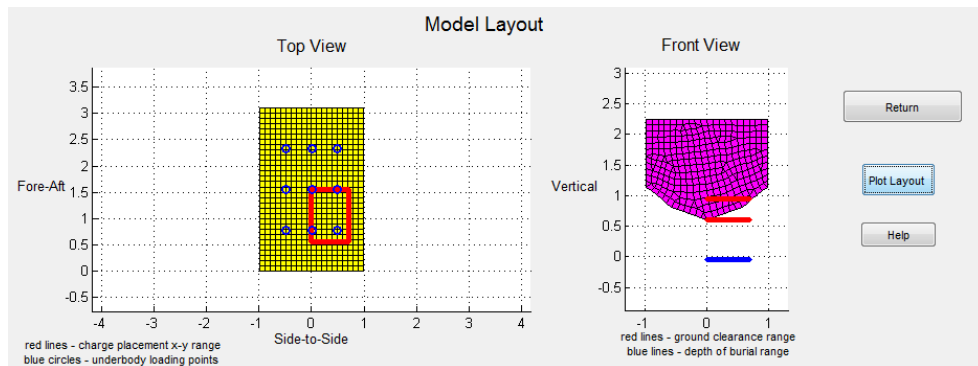


Figure 7: FRM applicable range and loading point grid for V-Hull case study, snapshot from FRM Build panel.

The ground clearance varies over a range of 0.65m. For the initial study, the size of the explosive and the depth of burial were held constant at Stanag Level 2 (in later simulations, the explosive size parameter was also allowed to vary and included in the meta-models) and 0.0508m, respectively. The loading point grid as well as the charge placement and depth of burial ranges can be seen in a snapshot from the FRM Build Panel, in figure 7.

The twenty training points were determined using the OSLH algorithm described previously. Figure 8 shows the x-y charge location for the training simulations used to construct the model. The two evaluation points discussed below are shown in red and labeled "1" and "2". It is important to remember that a uniform distribution of training points does not necessarily constitute a thorough and efficient survey of the design space. The OSLH algorithm provides a training point spectrum with good projection qualities that could not be intuited [21].

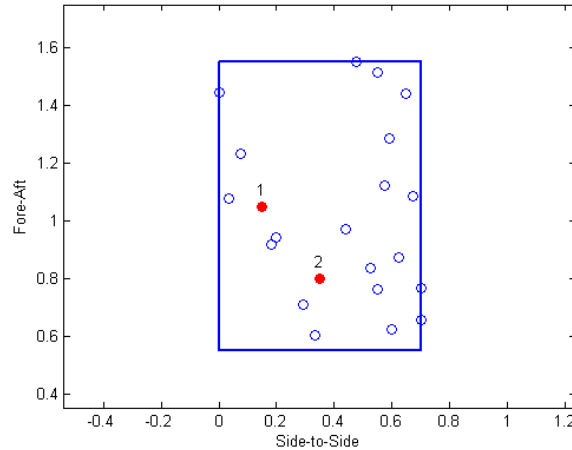


Figure 8: Training point (open blue circles) and evaluation point (solid red circles) locations.

Once the FRM has been constructed, it can be used to assess the pressure-time histories produced for hundreds of other explosive placement configurations which must be considered during survivability analysis. In order to demonstrate the accuracy of the FRM methodology, pressure time histories are evaluated at the two evaluation points, both by the FRM and by actual direct simulations. The input parameter combinations of the evaluation points are not part of the training point, or seed, set. Typical results for the reflected pressure histories at multiple locations on the TARDEC V-Hull are presented in figures 9-12. Pressure histories at various loading points along the structure are shown. The evaluation point associated with each plot is indicated by the filled red circle at which the red arrow points, and the loading point is indicated by the blue circle at which the blue arrow is pointed. The FRM applicable range for x and y charge placement is also outlined in red, with the location of the training points indicated by the filled blue circles. The FRM loading histories exhibit the same time dependency observed in the full simulation results, and peak values are similar. Thus, it is demonstrated that it is possible to replace time-consuming simulations with FRMs in underbody blast survivability studies.

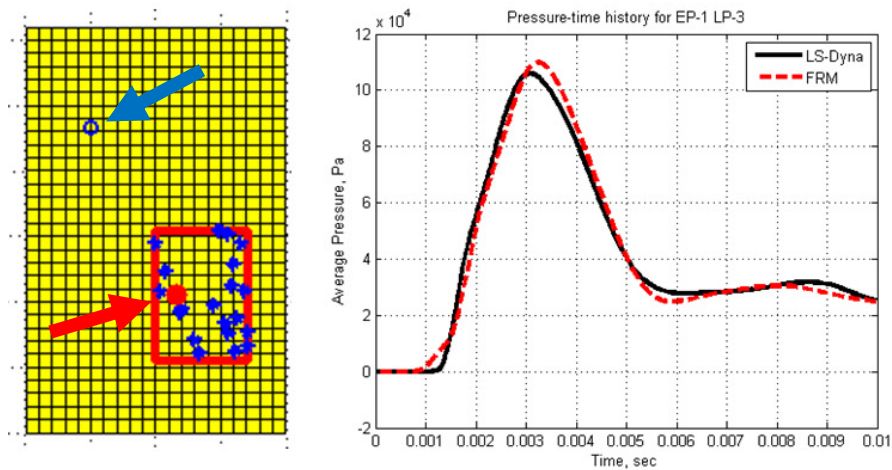


Figure 9: FEA-FRM comparison for Evaluation Point 1, Loading Point 3.

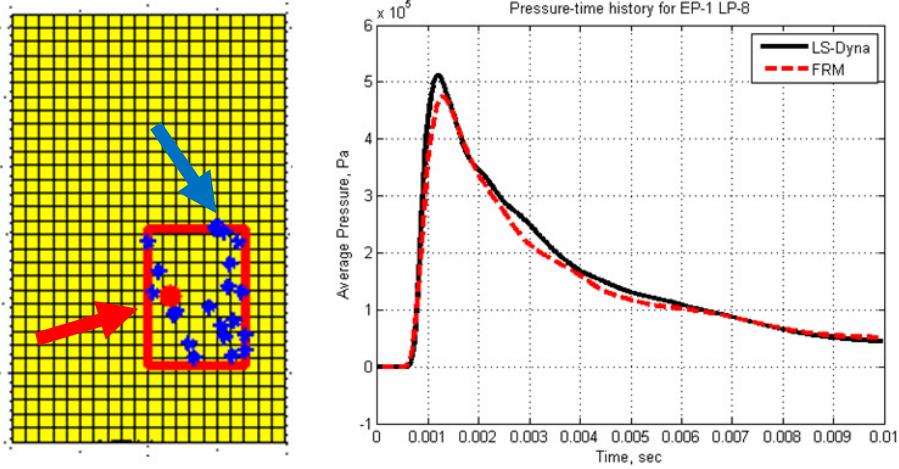


Figure 10: FEA-FRM comparison for Evaluation Point 1, Loading Point 8.

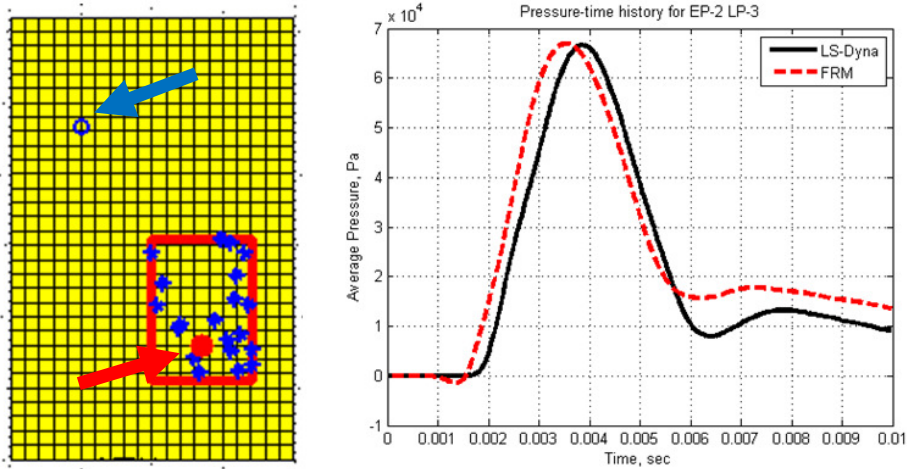


Figure 11: FEA-FRM comparison for Evaluation Point 2, Loading Point 3

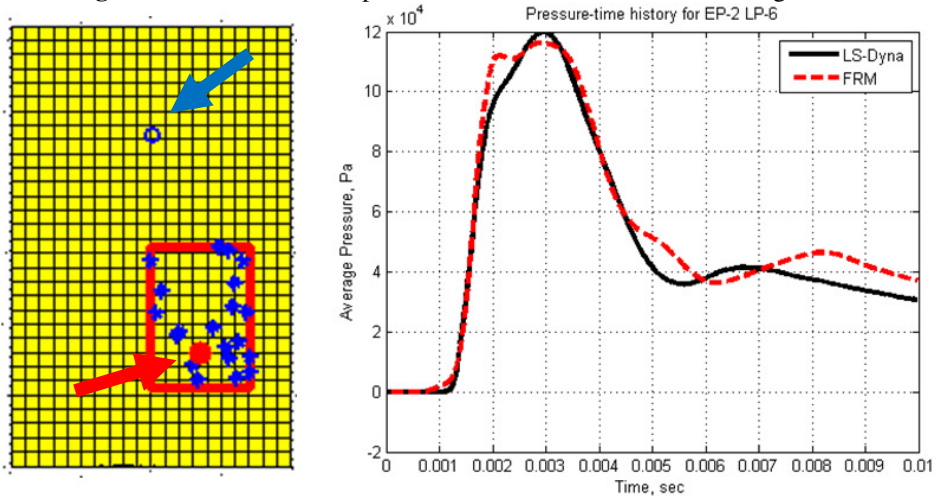


Figure 12: FEA-FRM comparison for Evaluation Point 2, Loading Point 6.

In addition to accurately computing pressure-time histories, as shown above, FRMs can also be used to directly predict the response of the vehicle (i.e. rigid body velocity, maximum acceleration, etc.) and/or occupant responses (forces, accelerations), without creating FRM models for the intermediary blast pressures acting on the vehicle underbody. To do so, the training point computations must include both the blast loading and the subsequent vehicle/occupant response. In this case, the vehicle/occupant response replaces the blast loading time histories as the input to the FRM creation process.

To demonstrate the effectiveness of FRMs for predicting structural response, metamodels have been constructed to predict peak vertical displacement of the vehicle underbody and average velocity over the roof region of the TARDEC V-Hull. Every underbody node (630 total) is used to calculate the maximum displacement of the hull bottom. Five nodes located on the top of the structure are used to calculate the maximum average velocity of the roof.

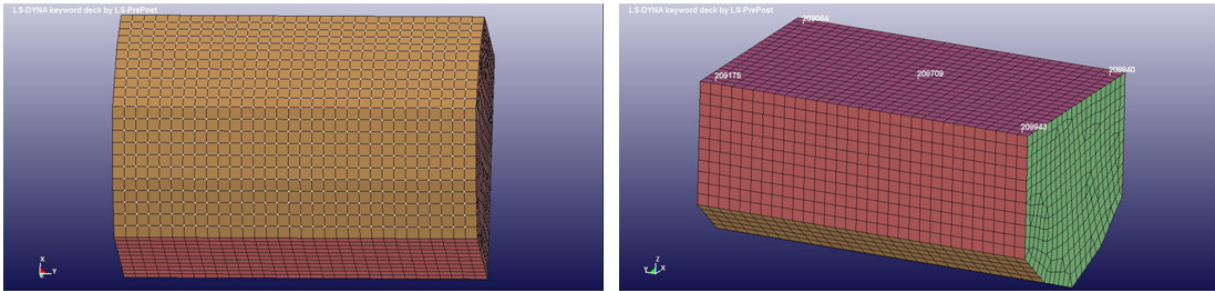


Figure 13: Underbody and roof nodes used to calculate the maximum displacement of the V-Hull bottom and the maximum average velocity of the V-Hull roof.

<p>Maximum Average Velocity \bar{V}_{Max} at One Surface of Hull: (Four Sides and Roof).</p> $V_j(t_k) = \sqrt{V_{xj}^2(t_k) + V_{yj}^2(t_k) + V_{zj}^2(t_k)} \quad (\text{jth Node at time step } t_k)$ $\bar{V}(t_k) = \frac{\sqrt{\sum_{j=1}^N V_j^2(t_k)}}{N} \quad (\text{N=5) at time step } t_k$ $\bar{V}_{Max} = \text{Max}[\bar{V}(t_k)]$
<p>Maximum Displacement u_{Max} at Bottom of Hull:</p> $u_j(t_k) = \sqrt{u_{xj}^2(t_k) + u_{yj}^2(t_k) + u_{zj}^2(t_k)} \quad (\text{jth Node at time step } t_k)$ $u(t_k) = \text{Max}[u_j(t_k)] \quad j = 1, \dots, N \quad (\text{N=630) at time step } t_k$ $u_{Max} = \text{Max}[u(t_k)]$

Figure 14: Mathematical approach for calculating vehicle response metrics.

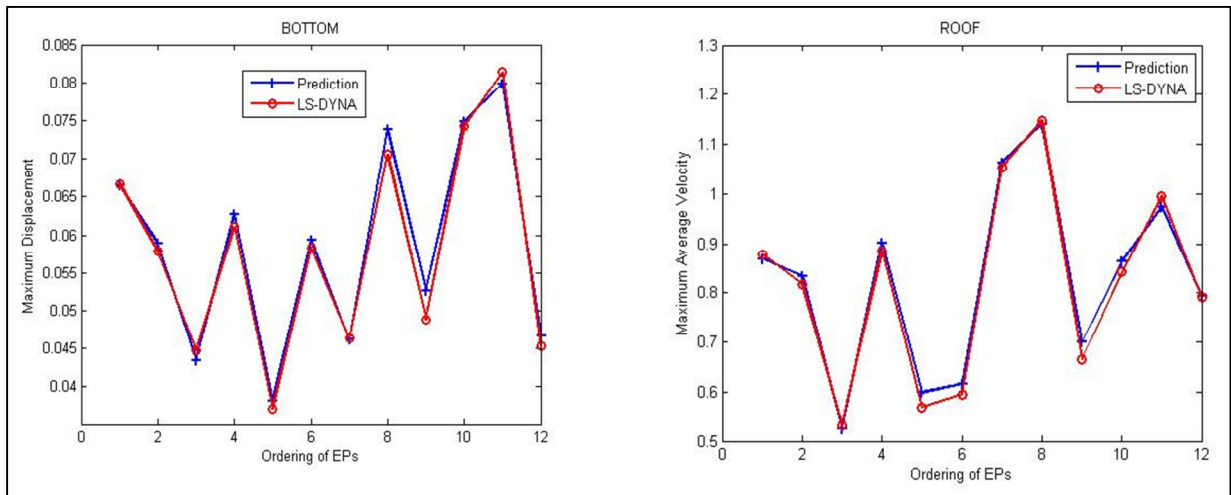


Figure 15: Representative comparison of structural responses evaluated by FRM and by the LS-DYNA solver for twelve evaluation points (EPs). The left plot depicts a comparison for maximum displacement of the V-Hull underbody. The right plot shows a comparison of the maximum average velocity of the V-Hull roof.

The nodes used for each analysis are shown in figure 13 and the mathematical methods of calculating each metamodel output are shown in figure 14. The results, shown in figure 15, again demonstrate excellent agreement with simulation data over the large number of 12 test/evaluation scenarios which were not included in the training point set.

CONCLUSIONS

The utilization of FRMs for Army vehicle survivability assessment enables rapid evaluation of an entire matrix of vehicle/explosive configurations. The results can be used to explore the design space in a systematic manner and to support informed decision-making. The quick turnaround time provides the necessary survivability information to the acquisition community when evaluating vehicle systems and conducting trade-off studies. Both the BEST process used to procure training point pressure histories and the FRM methodology have been validated for blast loading and structural response cases. The computational savings achieved by the FRM is significant, as FRM results are generated in a matter of seconds, compared with hours or days on high performance computing systems required for finite element simulations.

DISCLAIMER

Reference herein to any specific commercial company, product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the Dept. of the Army (DoA). The opinions of the authors expressed herein do not necessarily state or reflect those of the United States Government or the DoD, and shall not be used for advertising or product endorsement purposes.

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GLOSSARY

ALE	Arbitrary Lagrangian-Eulerian
APG	Aberdeen Proving Ground
ARL	Army Research Laboratory
ATD	Anthropomorphic Test Device
ATEC	Army Testing and Evaluation Command
BEST	Blast Event Simulation sysTem
COTS	Commercial Off-the-Shelf
CS/CSS	Combat Support & Combat Service Support
DoA	Department of Army
DoB	Depth of Burial
DTIC	Defense Technical Information Center, https://www.dtic.mil/
EP	Evaluation Points
FEA	Finite Element Analysis
FRM	Fast Running Models
MSU	Mississippi State University
OSLH	Optimal Symmetric Latin Hypercube
PCA	Principal Component Analysis
SimBRS	Simulation Based Reliability and Safety
SLAD	Survivability, Lethality and Analysis Directorate
SP/TP	Seed Points / Training points
TACOM	Tank Command
TARDEC	Tank Automotive Research & Development Engineering Center

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